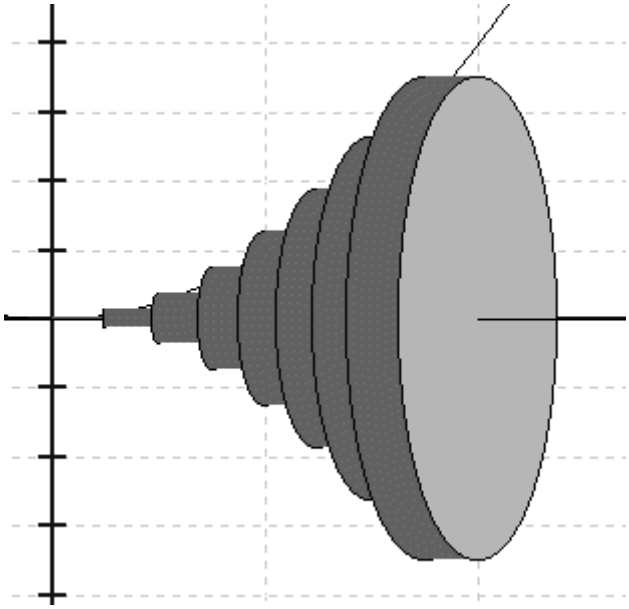
John Colet School

DEPARTMENT OF MATHEMATICS

Introduction to A level Maths



INDUCTION BOOKLET

SUMMER 2017

INTRODUCTION TO A LEVEL MATHS AT JCS

Thank you for choosing to study Mathematics in the sixth form at John Colet School. You will sit three papers at the end of year 13, containing questions on Pure Mathematics as well as Statistics and Mechanics. The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have prepared this booklet.

It is vitally important that you spend some time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics before you commence your course in September. You should have met all the topics before at GCSE. We will email a second booklet to you containing worked examples of the questions, look at this if you need support with any of the exercises.

We will test you at the start of September to check how well you understand these topics, so it is important that you have looked at all the booklet before then. This booklet is part of taking A Level Mathematics, it is not optional.

We hope that you will use this introduction to give you a good start to your A Level work and that it will help you enjoy and benefit from the course more.

Mr Stow

Exercise 1.1

1 Find the values of the letters p, q and r that make the following pairs of expressions always equal.

(a) $\frac{1}{7}x = \frac{x}{p}$ (b) $\frac{1}{5}(2x+3) = \frac{(2x+3)}{q}$ (c) $\frac{3}{10}(2-7x) = \frac{3(2-7x)}{r}$

2 Solve the following equations.

(a) $\frac{60}{x+4} = 12$ (b) $\frac{35}{2x-3} = 5$ (c) $\frac{20}{6-x} = \frac{1}{2}$

3 Make $\cos C$ the subject of the formula $c^2 = a^2 + b^2 - 2ab\cos C$.

4 (a) Multiply $\frac{x+5}{4}$ by 8. (b) Multiply $(x+2) \div 3$ by 12.

(c) Multiply $\frac{1}{2}(x+7)$ by 6. (d) Multiply $\frac{1}{4}(x-3)$ by 8.

5 Solve the following equations.

(a) $\frac{3}{4}(2x+3) = \frac{5}{8}(x-2)$ (b) $\frac{1}{6}(5x+11) = \frac{2}{3}(2x-4)$

(c) $\frac{5}{9}(3x+1) = \frac{7}{12}(2x+1)$

6 Make x the subject of the following equations.

(a) $\frac{a}{b}(cx+d) = x+2$ (b) $\frac{a}{b}(cx+d) = \frac{2a}{b^2}(x+2d)$

7 Simplify the following as far as possible.

(a) $\frac{a+a+a+a+a}{5}$ (b) $\frac{b+b+b+b}{b}$

(c) $\frac{c \times c \times c \times c \times c}{c}$ (d) $\frac{d \times d \times d \times d}{4}$

Exercise 1.2

1 Work out the following. Answers may be left as improper fractions.

(a) $\frac{4}{7} \times 5$ (b) $\frac{5}{12} \times 3$ (c) $\frac{7}{9} \times 2$ (d) $\frac{4}{15} \times 3$

(e) $\frac{8}{11} \div 4$ (f) $\frac{8}{11} \div 3$ (g) $\frac{6}{7} \div 3$ (h) $\frac{6}{7} \div 5$

(i) $\frac{3x}{y} \times x$ (j) $\frac{3x}{y^2} \times y$ (k) $\frac{5x^3}{4y} \div x$ (l) $\frac{5x^2}{6y} \div y$

(m) $\frac{5x^3}{2y} \times 3x$ (n) $\frac{3y^4}{4x^2z} \times 2x$ (o) $\frac{6x^2y^3}{5z} \div 2xy$ (p) $\frac{5a^2}{6x^3z^2} \div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$ (b) $V = \frac{4}{3}\pi x^3$ (c) $\frac{1}{2}(u + v) = tx$ (d) $W = \frac{2}{3}\pi x^2 h$

3 Simplify the following compound fractions.

(a) $\dfrac{\frac{1}{x} + 1}{\frac{1}{x} + 3}$ (b) $\dfrac{\frac{2}{x} + 1}{\frac{3}{x} - 1}$ (c) $\dfrac{\frac{1}{x+1} + 2}{\frac{1}{x+1} + 1}$

4 Write as single fractions.

(a) $\frac{2}{x-1} + \frac{1}{x+3}$ (b) $\frac{2}{x-3} - \frac{1}{x+2}$ (c) $\frac{1}{2x-1} - \frac{1}{3x+2}$ (d) $\frac{3}{x+2} + 1$

(e) $2 - \frac{1}{x-1}$ (f) $\frac{2x}{x+1} - 3$ (g) $\frac{3}{4(2x-1)} - \frac{1}{4x^2-1}$

5 Write as single fractions.

(a) $\frac{x+1}{\sqrt{x}} + \sqrt{x}$ (b) $\frac{2x}{\sqrt{x+3}} + \sqrt{x+3}$ (c) $\frac{x}{\sqrt[3]{x-2}} + \sqrt[3]{(x-2)^2}$

Exercise 1.3

1 Write without brackets.

(a) $(x + 5)^2$ (b) $(x - 4)^2$ (c) $(2x + 1)^2$

(d) $(3x - 2)^2$ (e) $(x + 2)(x - 2)$ (f) $(3x + 4)(3x - 4)$

2 Simplify the following equations into the form $ax + by + c = 0$.

(a) $(x + 3)^2 + (y + 4)^2 = (x - 2)^2 + (y - 1)^2$

(b) $(x + 5)^2 + (y + 2)^2 = (x - 5)^2 + (y - 2)^2$

(c) $(2x + 1)^2 + (y - 3)^2 = (2x + 3)^2 + (y + 1)^2$

3 Simplify the following where possible.

(a) $\sqrt{x^2 + 4}$ (b) $\sqrt{x^2 - 4x + 4}$ (c) $\sqrt{x^2 - 1}$

(d) $\sqrt{x^2 + 9x}$ (e) $\sqrt{x^2 - y^2}$ (f) $\sqrt{x^2 + 2xy + y^2}$

4 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 + 2x - 4$

(d) $x^2 - 4x - 3$ (e) $x^2 - 3x + 2$ (f) $x^2 - 5x - 6$

5 Write the following in the form $a(x + b)^2 + c$.

(a) $3x^2 + 6x + 7$ (b) $5x^2 - 20x + 17$ (c) $2x^2 + 10x + 13$

6 Write the following in the form $(ax + b)^2 + c$.

(a) $4x^2 + 12x + 14$ (b) $9x^2 - 12x - 1$ (c) $16x^2 + 40x + 22$

7 Factorise as fully as possible.

(a) $x^2 - 25$ (b) $4x^2 - 36$ (c) $4x^2 - 9y^4$

(d) $3x^2 - 7x + 2$ (e) $3x^2 - 5x + 2$ (f) $6x^2 - 5x - 6$

(g) $8x^2 - 2x - 15$

8 Multiply out and simplify.

(a) $\left(x + \frac{1}{x}\right)^2$ (b) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$ (c) $\left(x + \frac{2}{x}\right)\left(x - \frac{3}{x}\right)$

Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\dfrac{4 + 6x}{2}$ (d) $\dfrac{4 \times 6x}{2}$ (e) $\dfrac{3x + xy}{x}$

(f) $\dfrac{3x \times xy}{x}$ (g) $\dfrac{4x + 10y}{8x + 6y}$ (h) $\dfrac{3x - 6y}{9x - 3y}$

(i) $\dfrac{4x + 9y}{2x + 3y}$ (j) $\dfrac{4x + 6y}{6x + 9y}$ (k) $\dfrac{5xy + 6y^2}{10x + 12y}$

(l) $\dfrac{3x^2 + 4y^2}{6x^2 - 8y^2}$ (m) $\dfrac{x - 3}{3 - x}$ (n) $\dfrac{x^2 - 2xy - y^2}{y^2 + 2xy - x^2}$

2 Make x the subject of the following formulae.

(a) $\dfrac{ax}{b} = \dfrac{py}{qz}$ (b) $\dfrac{3\pi ax}{b} = \dfrac{4y^2}{qz}$

3 Simplify the following.

(a) $\dfrac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$ (b) $\dfrac{2\pi h^2}{rb} \div \frac{4}{3}\pi hr^2$

4 Simplify into a single factorised expression.

(a) $(x - 3)^2 + 5(x - 3)^3$ (b) $4x(2x + 1)^3 + 5(2x + 1)^4$

(c) $\frac{1}{2}k(k + 1) + (k + 1)$ (d) $\frac{1}{6}k(k + 1)(2k + 1) + (k + 1)^2$

5 Simplify as far as possible.

(a) $\dfrac{x^2 + 6x + 8}{x^2 - x - 6}$ (b) $\dfrac{3x^2 - 2x - 8}{x^2 - 4}$

(c) $\dfrac{(x + 3)^2 - 2(x + 3)}{x^2 + 2x - 3}$ (d) $\dfrac{x(2x - 1)^2 - x^2(2x - 1)}{(x - 1)^2}$

(e) $\dfrac{\dfrac{x^2}{\sqrt{x^2 + 1}} - \sqrt{x^2 + 1}}{x^2}$ (f) $-\dfrac{\dfrac{x}{2\sqrt{1 - x}} + \sqrt{1 - x}}{x^2}$

(g) $\dfrac{\dfrac{\sqrt{x}}{2\sqrt{1+x}} - \dfrac{\sqrt{1+x}}{2\sqrt{x}}}{x}$

(h) $\dfrac{\sqrt[3]{1+x} - \dfrac{x}{3\sqrt[3]{(1+x)^2}}}{\sqrt[3]{1+x}}$

Exercise 1.5

Solve the following simultaneous equations.

1 $x^2 + xy = 12$

$3x + y = 10$

2 $x^2 - 4x + y^2 = 21$

$y = 3x - 21$

3 $x^2 + xy + y^2 = 1$

$x + 2y = -1$

4 $x^2 - 2xy + y^2 = 1$

$y = 2x$

5 $c^2 + d^2 = 5$

$3c + 4d = 2$

6 $x + 2y = 15$

$xy = 28$

7 $2x^2 + 3xy + y^2 = 6$

$3x + 4y = 1$

8 $2x^2 + 4xy + 6y^2 = 4$

$2x + 3y = 1$

9 $4x^2 + y^2 = 17$

$2x + y = 5$

10 $2x^2 - 3xy + y^2 = 0$

$x + y = 9$

11 $x^2 + 3xy + 5y^2 = 15$

$x - y = 1$

12 $xy + x^2 + y^2 = 7$

$x - 3y = 5$

13 $x^2 + 3xy + 5y^2 = 5$

$x - 2y = 1$

14 $4x^2 - 4xy - 3y^2 = 20$

$2x - 3y = 10$

15 $x^2 - y^2 = 11$

$x - y = 11$

16 $\frac{12}{x} + \frac{1}{y} = 3$

$x + y = 7$

Exercise 1.6

1 Write the following as powers of x.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^5}$ (c) $\sqrt[5]{x}$ (d) $\sqrt[3]{x^5}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt{x^3}}$

2 Write the following without negative or fractional powers.

(a) x^{-4} (b) x^0 (c) $x^{1/6}$ (d) $x^{3/4}$ (e) $x^{-3/2}$

3 Write the following in the form ax^n.

(a) $4\sqrt[3]{x}$ (b) $\frac{3}{x^2}$ (c) $\frac{5}{\sqrt{x}}$ (d) $\frac{1}{2x^3}$ (e) 6

4 Write as sums of powers of x.

(a) $x^3\left(x+\frac{1}{x}\right)$ (b) $\frac{x^4+1}{x^2}$ (c) $x^{-5}\left(x+\frac{1}{x^2}\right)$

5 Write the following in surd form.

(a) $\sqrt{75}$ (b) $\sqrt{180}$ (c) $\frac{12}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{5}}$ (e) $\frac{3}{\sqrt{12}}$

6 Rationalise the denominators in the following expressions.

(a) $\frac{1}{\sqrt{2}-1}$ (b) $\frac{2}{\sqrt{6}-2}$ (c) $\frac{6}{\sqrt{7}+2}$

(d) $\frac{1}{3+\sqrt{5}}$ (e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

Exercise 2.1

1 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest $0.1°$.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

2 Solve the following equations for $-180 \leq x < 180$. Give your answers to the nearest $0.1°$.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

3 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest $0.1°$.

(a) $\sin 2x° = 0.829$ (b) $\cos 3x° = 0.454$ (c) $\tan 4x = 2.05$

(d) $\sin \frac{1}{2}x° = 0.8$ (e) $\cos \frac{1}{2}x° = 0.3$ (f) $\tan \frac{1}{3}x° = 0.7$

Exercise 2.2

Do not use a calculator in this exercise.

1 In this question θ is in the range $0 \leq \theta < 90$.

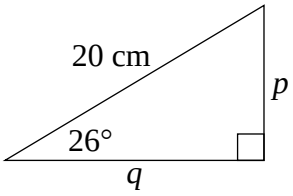
(a) Given that $\sin\theta = \frac{12}{13}$, find the exact values of $\cos\theta$ and $\tan\theta$.

(b) Given that $\tan\theta = \frac{6}{7}$, find the exact values of $\sin\theta$ and $\cos\theta$.

(c) Given that $\cos\theta = \frac{5}{8}$, find the exact values of $\sin\theta$ and $\tan\theta$.

2 Find expressions, of the form $a\sin\theta$ or $b\cos\theta$, for the sides labelled with letters in these triangles.

(a)



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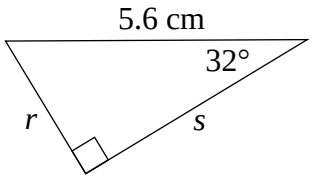

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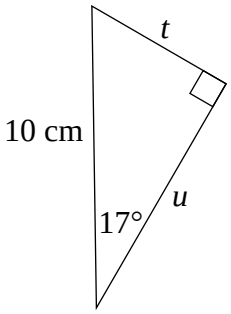

(d)